Uncertainty estimation for comb based laser calibrations by direct comparison of 3 different combs

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Abstract

We present a GUM conform treatment for estimating the measurement uncertainty of laser frequency calibrations using a femtosecond optical frequency comb generator. Using the formalism developed a schema for a comb-comb comparison is proposed which delivers an upper bound for the uncertainty component originating from the calibration method. Experiments using three different combs are presented and the uncertainties analyzed. This work led to a significant reduction of the uncertainties for comb based laser calibrations for both institutes involved. To our knowledge this is the first direct comparison of combs in the mode of their actual intended operation as calibration instruments.

1 Introduction

Tracing dimensional metrology measurements back to the SI definition of the metre is nowadays done with the help of optical femtosecond frequency comb generators (denoted by "combs" in the following). These instruments are used to calibrate the frequency of stabilized laser sources which at the same time gives the vacuum wavelength with the same relative uncertainty. This calibration constitutes the first step in the traceability chain for interferometric measurements and it is offered as a service to the public by many NMIs. Under the CIPM MRA [1] all calibration services which are recognized by other participating countries and accredited bodies have to fulfil certain conditions – one of the most important is a valid measurement uncertainty budget which eventually induces reviewed CMCs to be published.

In the field of comb based calibrations relatively few CMC entries have been treated so far and the rules for review are by no way consolidated. The uncertainty claims span the whole spectrum from very low (1×10^{-16} not including the reference) up to very conservative values (1×10^{-9}) which clearly give away calibration possibilities.

The first group justifies their claims by specialized experiments which prove the consistency of the underlying comb principle. One should note that these setups are quite different from an ordinary laser calibration; even the combs may be different [2].

The conservative claims compare actual laser calibrations using different combs or other calibration techniques. The reproducibility of common transportable lasers limits the uncertainty of this comparison to about 1 kHz (2×10^{-12} in relative), which is worse than real uncertainty of combs and reference RF (radio frequency) signals (but definitely sufficient for dimensional measurements).

In this contribution we present a relative simple way to overcome this dilemma. The proposed scheme is a special kind of comb-comb comparison while mimicking the actual laser calibration protocol of both parties. The comb based uncertainty component can be separated from other contribution (from RF-reference and laser under test, respectively) by skilful use of correlations. While correlations can be a kind of nuisance in other uncertainty calculations, here they are a vital part for the technique proposed.

2 Measurement uncertainty for comb based laser calibration

Every GUM compliant uncertainty calculation must start with the definition of the measurand and the formulation of the model equation for it ([3] clause 4.1). In frequency metrology special mathematical tools have been developed to deal with time series of correlated data. Also the use of relative uncertainties is common practice in this field. However in our case a more traditional treatment is appropriate. We simply start with the comb equation which relates the laser frequency (the measurand) with the repetition f_{rep} and offset f_{off} frequencies of the comb, and the beat frequency f_{beat} with the n-th comb mode, respectively [4,5]. Without loss of generality a specific choice of signs is chosen here

$$f_{\mathrm{Laser}}\left(t\right) = n f_{\mathrm{rep}}\left(t\right) - f_{\mathrm{off}}\left(t\right) - f_{\mathrm{beat}}\left(t\right) - \lambda_{\mathrm{r}} - \lambda_{\mathrm{p}} \tag{1}$$

In (1) two quantities are added which account for uncertainty components of a real laser artefact (as opposed to just the laser frequency). Here λ_{p} combine all working parameter specific influences of the laser while λ_{r} models all other (random) influences. In this context one should make clear that all quantity symbols actually represent mean values (samples) over a given gate time τ .

$$f\left(t\right) = \int_{t}^{t+\tau} f^{\mathrm{inst}}\left(t\right)\mathrm{d}t \tag{2}$$

The actual measurand is usually the mean value over a given period of time (integration time, also designated by τ) using samples as defined in (2). To simplify notation we will not differentiate between these cases and the reference to t and τ will be dropped in the following except where there is danger of confusion. There are at least two different ways to access the two characteristic frequencies of a comb, this can be either the direct measurement or, more common, they are locked to a stable reference frequency. The model equation and the uncertainty will reflect the specific choice. In the following derivation we will discuss the locking route where both, the repletion frequency and the offset frequency, are locked to a multiple of a stable input frequency f_{in} . The deviations of the ideal locking behaviour are modelled by γ_{rep} and γ_{off} , respectively, while any instability and deviations of the input frequency are handled by the quantity ρ .

$$f_{\mathrm{rep}} = k_{\mathrm{rep}} f_{\mathrm{in}} + \gamma_{\mathrm{rep}} \tag{3}$$

$$f_{\mathrm{off}} = k_{\mathrm{off}} f_{\mathrm{in}} + \gamma_{\mathrm{off}} \tag{4}$$

$$f_{\mathrm{in}} = f_0 + \rho \tag{5}$$

Equations (3) and (4) do not imply that the repetition or offset frequency is in reality obtained by a simple multiplication process, the actual technique can be quite complex involving servos, synthesizers, PLLs, etc. but the knowledge about these processes is covered by γ_{rep} and γ_{off} , respectively. To quantify them is in general a quite laborious task and sometimes not even possible.

For the beat frequency we separate the influence of the reference frequency from other counter specific errors γ_{cnt} . It includes contributions from the signal to noise ratio of the beat signal, the counter resolution, but also a specific data treatment like automatic outlier removal etc. The influence of the reference frequency is separated from this contribution by the explicit inclusion of the input frequency f_{in} . This is important in the later separation of the three main uncertainty sources of comb measurements. The actual beat frequency is separated in a constant part denoted by f_{b} and the small variable part λ_{b} (which carries the actual measured information together with the related uncertainty). The constant value of f_{b} is arbitrary, especially if we look at a single data point at time t. It is convenient to set it to the mean value of the measured beat frequency, in this way the expectation value of λ_{b} becomes zero and the following treatment will be much clearer.

$$f_{\mathrm{beat}} = f_{\mathrm{b}} \frac{f_{\mathrm{in}}}{f_0} + \lambda_{\mathrm{b}} + \gamma_{\mathrm{cnt}} = f_{\mathrm{b}}\left(1 + \frac{\rho}{f_0}\right) + \lambda_{\mathrm{b}} + \gamma_{\mathrm{cnt}} \tag{6}$$

Collecting everything and substituting into (1) one ends up with the model equation for a laser calibration using a comb.

$$f_{\text{Laser}} = \left(\left(n\,k_{\text{rep}} - k_{\text{off}} \right) f_0 - f_{\text{b}} \right) + \frac{\left(n\,k_{\text{rep}} - k_{\text{off}} \right) f_0 - f_{\text{b}}}{f_0} \rho + n\,\gamma_{\text{rep}} - \gamma_{\text{off}} - \gamma_{\text{cnt}} - \lambda_{\text{b}} - \lambda_{\text{s}} - \lambda_{\text{p}} \qquad (7)$$

One should note that with the exception of ρ, γ_i and λ_i all quantities in (7) are constants, hence do not contribute to the uncertainty. Keeping in mind that the first bracket term is actually the (nominal) laser frequency $f_{\text{Laser}}^{\text{N}}$, (7) can be simplified considerably.

$$f_{\text{Laser}} = f_{\text{Laser}}^{\text{N}} + \frac{f_{\text{Laser}}^{\text{N}}}{f_0} \rho + \gamma + \lambda \qquad (8)$$

In (8) all contributions related to the comb set up are subsumed in γ, while contributions coming from the laser standard are denoted by λ (a change in signs is included in this symbols). In the derivation of the model equation care was taken to avoid correlations between the input quantities. Since covariances between the input quantities are unlikely, the uncertainty of the measurand is simply the quadratic sum of the input uncertainties.

$$u^2\left(f_{\text{Laser}} \right) = \frac{\left(f_{\text{Laser}}^{\text{N}} \right)^2}{f_0^2} u^2(\rho) + u^2(\gamma) + u^2(\lambda) \quad \left\{ = u_{\text{comb}}^2 + u^2(\lambda) \right\} \qquad (9)$$

The first two terms make up the uncertainty of the comb setup and are relevant for CMC claims. The last contribution originates from the laser to be measured only and is not of direct interest in this context.

2.1 Estimation of u_{comb} by theoretic calculation

In principle one can evaluate the various contributions leading to the uncertainty u_{comb} from one's knowledge on the setup and all influence quantities. This is straightforward for the very first term in (9) since it is dominated by the properties of the reference frequency.

A comb can be referenced either to local RF/microwave frequency standards, i.e. to the NMI's realization of the SI second (compared via the key comparison CCTF-K001.UTC [6]), or to an atomic or molecular frequency from the list of recommended radiations for the realization of the meter and other optical frequency standards [7]. The comb may alternatively be referenced to an RF oscillator disciplined by timing signals from a global navigational satellite system (GNSS) such as GPS, Galileo, or GLONASS. The GNSS timing signal is traceable to the SI second and hertz via the system's primary timing facility, e.g. for GPS, via UTC(USNO), but the uncertainty in the derived frequency must be carefully evaluated as described e.g. in [8].

The second term in (9) is the challenging uncertainty contribution which is not easily assessable. For an analysis of $u(\gamma)$ the specific contributions given in (7) have to be studied. This is in general a non trivial task and not topic of this paper.

2.2 Estimation of u_{comb} using a calibrated laser source

An obvious technique is to use an elsewhere calibrated laser (with given uncertainty) for calibration of the comb to be investigated. Here this laser can be considered as a standard and it is used to assess the combined comb-related influence quantity γ. Thus one arrives at the following model equation.

$$\gamma = f_{\text{Laser}}^{\text{calib}} - f_{\text{Laser}}^{\text{N}} - \frac{f_{\text{Laser}}^{\text{N}}}{f_0} \rho - \lambda \qquad (10)$$

Where the associated uncertainty is:

$$u^2(\gamma) = u^2\left(f_{\text{Laser}}^{\text{calib}}\right) + \frac{\left(f_{\text{Laser}}^{\text{N}}\right)^2}{f_0^2} u^2(\rho) + u^2(\lambda) \tag{11}$$

As long as the uncertainty of both, the calibrated laser frequency and $u(\lambda)$ are considerable small, one obtains a viable estimation for $u(\gamma)$. Together with the uncertainty originating from the RF-reference which can be estimated as discussed in the former section one gets eventually u_{comb}.

Although laser sources with the desirable properties exist, they are not very common and transportable ones are very rare. Other laser sources, like iodine stabilized He-Ne-lasers, are quite widespread at the NMI level, but the uncertainty of these devices is in the order of a few kHz. By this technique the true uncertainty of the comb contribution is thus masked by the properties of the laser. Up to now this way is only open for a limited number of institutes and works for a few selected wavelengths only.

2.3 Estimation of u_{comb} by direct comparison of two combs

Like in other metrology fields a comparison of two measuring instruments (in this case two combs) can be used to check the consistency of the uncertainty claims. In cases where the respective uncertainties are not known before, the uncertainty of the comparison can be used as an upper bound. The influence of the reference frequency should be excluded since it can be characterized by other means. For this end care must be taken to fulfill some important conditions.

The most important point is a strict equivalence between the setup and procedures when the comb is used for calibration and for the comb-comb comparison. It is relatively easy to compare critical parts of combs by task specific setups. Examples include consistency of the comb mode distribution by using fundamental and frequency converted light [5], optical frequency ratios [9], and laser source locking [10].

Whilst such experiments are of great importance for the thorough understanding of one’s setup, they can not immediately be used for a determination of u_{comb} or $u(\gamma)$. Obviously the two cases deal with different measurands and uncertainties for different measurands are incomparable. Moreover for these experiments the combs must usually be operated in a different way, thus compromising insights for the “ordinary” use. The results of such experiments can however constitute valuable information for a theoretical estimation as exemplified in section 2.1.

Here we propose a scheme where two combs can be compared while they are operated in a way like for “ordinary” laser frequency calibration. Even more the influence of the RF-reference cancels out in our setup so that the respective uncertainty component can be handled separately. No modification on any of the two combs is necessary (however a synchronization of all counters is helpful to minimize the comparison uncertainty).

The two combs to be compared must be near to each other so that the same laser radiation can be measured by them at the same time. Ideally the light to be measured is generated by a frequency stabilized laser and transferred via a beam splitter to both combs. Similarly the same RF-reference frequency should be provided to both instruments. In cases where the combs demand different input frequencies (hence a distribution amplifier can not be used) they should be derived from the same standard (clock). Practically the setup requires the two combs to be in close proximity. For comparison between NMIs at least one comb (but not its RF-source) must be transportable.

When the two combs are referenced by the same input frequency source f_{in} and both measure the same laser f_{Laser} then many of the influence quantities will cancel out. Provided both combs also use the same gate time for their frequency counters and are moreover adequately synchronized we get at each moment (indices 1 and 2 label quantities obtained by comb 1 and comb 2, respectively):

$$\Delta f_{12} = f_{\text{Laser},1}^{\text{N}} - f_{\text{Laser},2}^{\text{N}} + \frac{f_{\text{Laser},1}^{\text{N}} - f_{\text{Laser},2}^{\text{N}}}{f_0} \rho + \gamma_1 - \gamma_2 + \delta \tag{12}$$

where $\Delta f_{12} \equiv f_{\mathrm{Laser},1} - f_{\mathrm{Laser},2}$ denotes the difference of the laser frequency as measured by comb 1 and comb 2, respectively. The new introduced quantity δ models setup specific asymmetries which are not covered by (8). Typically errors in the synchronization of the counters or differences in the light path and signal delay times will be covered so. It is always possible to choose $f_{\mathrm{b},1}$ and $f_{\mathrm{b},2}$ in a way so that $f^{\mathrm{N}}_{\mathrm{Laser},1} = f^{\mathrm{N}}_{\mathrm{Laser},2}$. Note that according to (6) the f_{b} is just a constant which can be chosen such that the convenient notation is reached. This very choice actually defines the nominal laser frequency. By doing so the influence of the reference frequency disappears and equation (12) simplifies to:

$$\Delta f_{12} = \gamma_1 - \gamma_2 + \delta \tag{13}$$

The respective uncertainties obey (14).

$$u^2(\gamma_1) + u^2(\gamma_2) = u^2(\Delta f_{12}) - u^2(\delta) \tag{14}$$

This is the principal relation between the demanded uncertainties of the combs and quantities measured according to the proposed schema. Obviously there are three problems with this formulation:

1. Only the combined uncertainty of both combs is accessible with this technique. It is not possible to obtain the actually sought uncertainty for a single comb. With a pair wise comparison of at last three combs this would be possible however (three cornered hat method [11]). In the absent of this option (14) is still valuable since it gives an upper bound for each comb's uncertainty. This is sufficient in most cases and will be applied also in this paper.

2. The uncertainty $u(\delta)$ has to be estimated in order to get the left hand side of (14). It is prudent to set it to zero, since in this way again an upper bound of the comb's uncertainties is estimated. A more detailed discussion of this point will follow later.

3. In order to evaluate (14) one needs a way to derive $u(\Delta f_{12})$ from the measurement data. Since the data is recorded as a time series, it is tempting to use the empirical sample standard deviation σ_τ as a measure for this uncertainty. Whether the standard deviation is a suitable measure for the uncertainty has to be checked in each case. Specifically the deviation of $\overline{\Delta f_{12}}$ from 0 must be consistent with its uncertainty for all τ considered. In phase-stabilized signals (white phase noise, which are relevant here) one can alternatively use the Allan standard deviation σ^{A}_τ which is proportional to the former one [12]. The validity of the proportionality between the standard and the Allan deviation should be checked experimentally. The value of this ratio can give indication on the contribution of $u(\delta)$ since it will change with the noise type. Also a significant drift in Δf_{12} could be detected in this way.

For the uncertainty of a single comb we obtain eventually the following inequality which will be used in the analysis of the experimental data.

$$u_\tau(\gamma_1) \le \sigma_\tau(\Delta f_{12}) \text{ and } u_\tau(\gamma_2) \le \sigma_\tau(\Delta f_{12}) \tag{15}$$

It is very important to keep in mind that (15) is implicitly dependent on all operating conditions of the comb. From those the total integration time τ is the most important one. It is indicated as an index in (15).

3 Experimental

3.1 Setup

Three different combs which are normally operated as calibration facilities at their home labs (BEV and CMI), were used for this work. The main characteristics of the instruments can be found in the following table.

Table 1: Compilation of relevant parameters of the three combs studied

Comb #	1	2	3
Designation	BEV fibre	CMI Ti:Sa	BEV Ti:Sa
fs-laser	Er-doped fibre, ring resonator	Ti-sapphire, linear resonator	Ti-sapphire, ring resonator
Type	frequency doubled comb radiation used for experiment	2 separate PCFs	common PCF (for experiment and for nonlinear interferometer
f_{rep}	250 MHz	200 MHz	1 GHz
f_{off}	20 / 40 MHz	20 MHz	40 MHz
f_{synth}	$4 \cdot f_{\text{rep}} - 980\,\text{MHz}$	$1\text{GHz} - 5 \cdot f_{\text{rep}}$	$f_{\text{rep}} - 970\,\text{MHz}$

Although all instruments were designed and manufactured by the same company [13], the specific setups for repetition and offset frequency control are constructed in different ways. Furthermore many optical and electronic components as well as the software were modified by the respective institutes. Probably the only common parts of the three combs are the frequency counters. Each of the instruments utilise 4 channel synchronous digital phase recorders (instrument name: FXM [14, 15]) which are operated as death time free frequency counters with a gate time of 1 s. The counters can be concatenated to an in principle unlimited number of channels. Here we take use of this option to synchronously record the data of the two combs compared. By doing so one of the combs does not need any software modification, in fact it can be used as in everyday work while still transferring the data to the second comb. The software of this second comb however must be capable to record and process all 8 channels while still doing "ordinary" measurements with its own 4 channels.

Usually each comb measures at least the beat frequency, the down mixed repetition frequency (denoted by f_{synth} in table 1) and the offset frequency, respectively. The later two may be used to compute the instantaneous comb mode frequency, or – more often – to check the working conditions of the respective locks. At both, the BEV [16] and CMI [17] the fourth channel is in addition used to check the validity of the beat signal.

Comb 1 was compared with comb 2 at the site of the CMI, comb 1 with comb 3 at the BEV. It was not possible to perform the last comparison (comb 2 vs. comb 3) in the course of this work since none of them could be transported without possible change in their characteristics.

None of the combs in table 1 is a dedicated portable instrument; however the BEV fibre comb is compact enough for a transport by a pickup truck. The optics part is mounted on a small mobile optical table, while a single rack contains the complete control electronics. This comb was transported from Vienna to Prague (distance approximately 350 km) for the comparison with the CMI Ti:Sa comb in November 2008. Astonishingly the comb not only survived the vibrations but also a night near freezing temperature and the subsequent water condensation at its surfaces.

The principal setup for a comb comparison according to the proposed schema can be seen in Fig. 1. The two combs are placed near each other in the same laboratory room but on different tables. The output of a single laser is subdivided into two beams by a variable beam splitter. Each of the two beams is transferred to the respective comb. Since the two combs are located on different optical tables it is difficult to use free space beams therefore the use of at least one monomode optical fibre is indicated. The laser source must be of a type so that both combs can actually measure its frequency. This requires a single frequency and moderately stabilized laser. Additionally a high output power greatly simplifies the alignment efforts. Two type of HeNe lasers (633 nm) were used in this experiment: a polarisation stabilised laser with an output power of about 1 mW and the iodine stabilized standard PLO3 [18] with an output power of about 80 µW. Moreover the later laser challenges the comb measurement by it's strongly frequency modulated light.

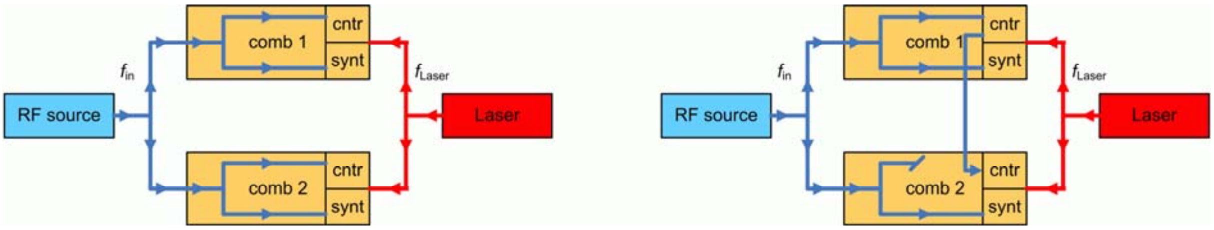

Figure 1: Schematic setup for a comb-comb comparison. Left setup without synchronisation, right with counter synchronisation. cntr: counter, synt: RF-synthesizer. RF signals are denoted by blue lines and f_{in}, optical signals by red lines and f_{Laser}.

To provide the same reference frequency (RF) to the instruments offers no special problem. All instruments in these experiments need a single input of 10 MHz which is internally distributed to the various synthesizers and counters. At the site of CMI a GPS-disciplined Rb-oscillator, at the BEV an active H-maser was used.

For the estimation of the uncertainty of a comb setup the specific data analysis is a vital part. It is very important that it is not altered because of framework requirements of the comparison. This condition is in principle fulfilled as outlined above since only the BEV fibre comb has modified software which is capable to additionally record the frequency readings of the second comb. This very software is also used in standard calibrations. The software of the second instrument needs not be changed at all.

However to ease the analysis a special off line software was used where the frequency as measured by the two combs could be compared at any given time. Two aspects are especially important in this context: the automatic detection of blunders (outliers) and numeric issues. In all instruments considered, blunders in f_{rep} and f_{off} are detected by checking the measured deviations from their preset values. For the detection of blunders in f_{beat} both institutes use the technique as described in [16] where the beat frequency is measured simultaneously with two counter channels of different bandwidths. The respective tolerances are specific for the combs but held constant during the experiments. Only data points recognized as valid by both combs are used for the final analysis.

The equivalence of this off line program with the original software needs an extra consideration. Calculating the laser frequency according to (1) can bring up subtle numerical errors due to round-off errors, arithmetic overflow and arithmetic underflow. For this purpose the mean laser frequency as calculated by the original program and the off line program was compared where care must be taken to use exactly the same number of data points. Equivalence was assumed when the two results did not differ by more than 2 mHz for the approx. 470 THz values. (Note this refers only to the output of two different programs with the exact input data, not to the result of the two combs!)

3.2 Results

Fig. 2 shows the result of a comparison between the BEV fibre comb and the CMI Ti:Sa comb, respectively. The inset shows the laser frequency of the SIOS laser as measured by the two instruments. In the scale used the difference between the instruments is indiscernible. The lower part shows the difference between these two measurements Δf_{12}, now with a different scale. The quantity of interest is the uncertainty $u(\Delta f_{12})$, estimated as the sample standard deviation σ_τ for different integration times τ. Fig. 3 shows a plot of σ_τ together with the Allan standard deviation σ_τ^{A} for both comb combinations studied. The higher uncertainty of the comb BEV Ti:Sa as compared to the other two is obvious. This is probably caused by the worse tracking capabilities of this instrument but the exact cause is not important for the uncertainty estimation. According to (15) one has just to use the found standard deviation.

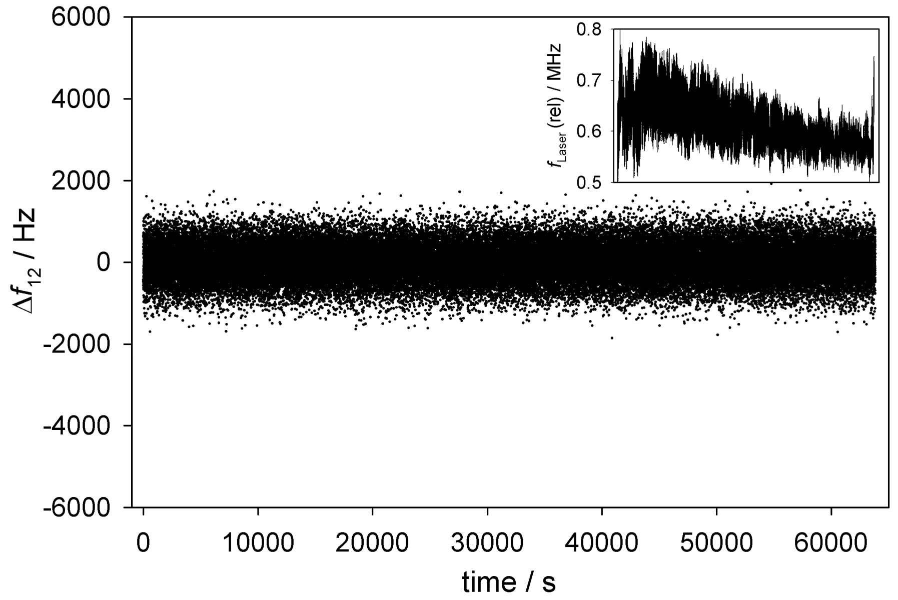

Figure 2: Deviations of individual 1 s samples between the two combs CMI Ti:Sa and BEV fibre. The inset shows the actual laser frequency relative to an offset 473 612 521 MHz.

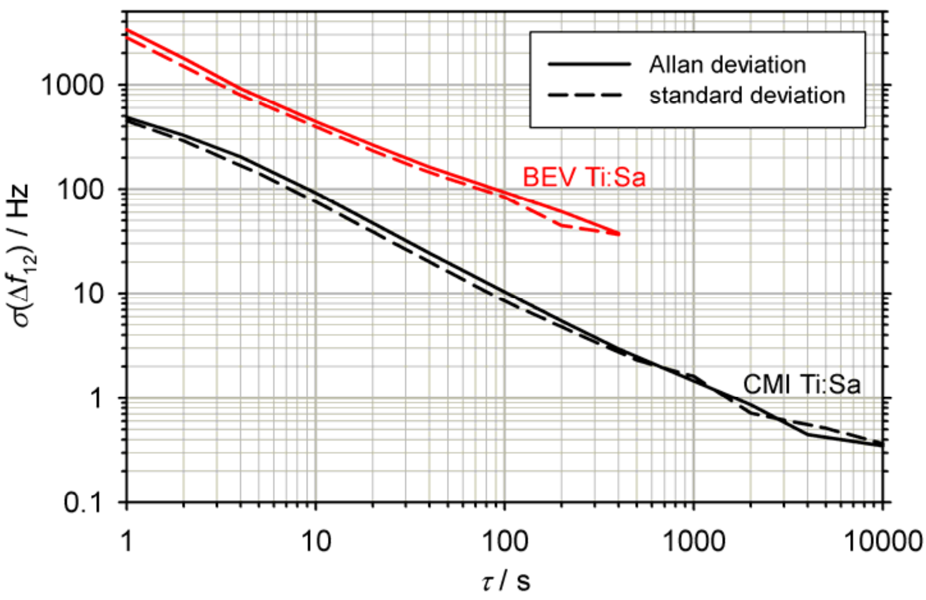

Figure 3: Allan standard deviation (solid lines) and sample standard deviation (dashed lines) for the frequency difference between BEV fibre comb and the two other combs considered.

Although not necessary for our purpose it is interesting to estimate the influence of $u(\delta)$ on the result. As mentioned above the main contribution originates from asymmetries in the delay of light paths and signal lines. In the current setup the laser light is transferred to the first comb as a free space beam while it is transferred to the second comb via a 20 m long optical fibre. The counters are synchronized using 3 m long BNC cables and the reference frequency is provided by cables of nominally equal length. So the delay can amount about 100 ns. For the investigation of this effect it is tempting to use a laser with fast frequency fluctuations. Here we used an iodine stabilized laser (frequency modulated with a modulation width of 6 MHz dithered with 1.11 kHz). The actual experiment is not that easy since the output power of the laser PLO3 is quite low. Table 2 summarizes the results of all measurements during this comparison. No significant difference can be observed between the two different lasers.

Table 2: sample standard deviation, Allan standard deviation (both for τ = 1 s), mean frequency difference, standard deviation of the mean and number of valid data points.

comb 1	comb 2	Laser	$\overline{\Delta f_{12}}$ / Hz	$\sigma_{1\text{s}}$ / Hz	$\sigma_{1\text{s}}^{\text{A}}$ / Hz	$\sigma_{n\text{s}}$ / Hz	n
CMI Ti:Sa	BEV fibre	SIOS	–25.2	387	421	7.3	2 818
CMI Ti:Sa	BEV fibre	SIOS	–12.6	354	389	8.4	1 882
CMI Ti:Sa	BEV fibre	SIOS	–5.1	410	447	3.7	12 204
CMI Ti:Sa	BEV fibre	SIOS	–0.5	471	510	3.8	15 794
CMI Ti:Sa	BEV fibre	SIOS	+0.7	448	485	1.8	63 583
CMI Ti:Sa	BEV fibre	PLO3	+6.9	477	517	9.8	2 356
CMI Ti:Sa	BEV fibre	PLO3	–2.9	475	511	5.8	6 642
BEV Ti:Sa	BEV fibre	SIOS	+55.1	5990	7250	165	1315
BEV Ti:Sa	BEV fibre	SIOS	–51.6	4350	5200	50	7654
BEV Ti:Sa	BEV fibre	SIOS	+124	6040	7170	241	631
BEV Ti:Sa	BEV fibre	SIOS	+103	5390	6180	161	1116

But also when a strict synchronisation is not practical (when the two combs use different counters or incompatible software) it is still possible to compare them. In this case the measurements on the two combs must be started and stopped manually as coinstantaneous as possible. In this way the correlation originating from the same reference and the same laser source is maintained for the mean value of Δf_{12} to a high degree. Unfortunately it is not straightforward to estimate the actually needed $u\left(\Delta f_{12}\right)$ by a single run. In principle one can repeat the experiment a number of times for a given integration interval and calculate the sample standard deviation. This was proposed already in [19] but obviously this is a very time consuming task which can not be recommended in general. But also from a single measurement on can get a feeling for the performance of the combs. As presented in table 3 Δf_{12} = 90 Hz for the two mode stabilized laser with an integration time of 1340 s.

Table 3: Result of an unsynchronized comparison. Integration time 1340 s.

Comb	Measured laser frequency
CMI Ti:Sa	473 612 521 610 094 Hz
BEV fibre	473 612 521 610 004 Hz

3.3 Mode of operation

In comb based laser calibrations it is common practice to keep f_{rep} constant during the measurement. For the data evaluation the preset value is used in equation (1) for all valid data points. This is the procedure which was applied in this study also. In special cases a repetition frequency might be locked to an other parameter (e.g. phase signal of a laser interferometer) or it might be even free running. In such cases the repetition frequency has to be determined during the measurement.

As a side result we can also check the behaviour of the two combs when f_{rep} is not considered to be predetermined but actually measured (this is the standard technique used at CMI). Since all relevant counter readings are recorded anyway this is just a matter of offline analysis. However in this case the repetition frequency remains locked, failing to do so will be beyond the scope of our measuring scheme. The measurement model (1) for the uncertainty estimation remains formally unchanged only (3) should be modified to take account of the counter related uncertainties. This would however only be necessary if a full *ab initio* uncertainty calculation is wanted. Within our scheme the complete comb related uncertainty is obtained and knowledge on internal details are not necessary at all. The evaluation was performed per comb in two different ways: repetition frequency is determined just by the preset value of the relevant synthesizer (i.e. fully relying on servo, in following marked by "S"), and repetition frequency is measured by the relevant counter (marked by "M"). In Fig. 4 and Table 4 we compile typical results for the dispersion of 1 s samples for all four different combinations.

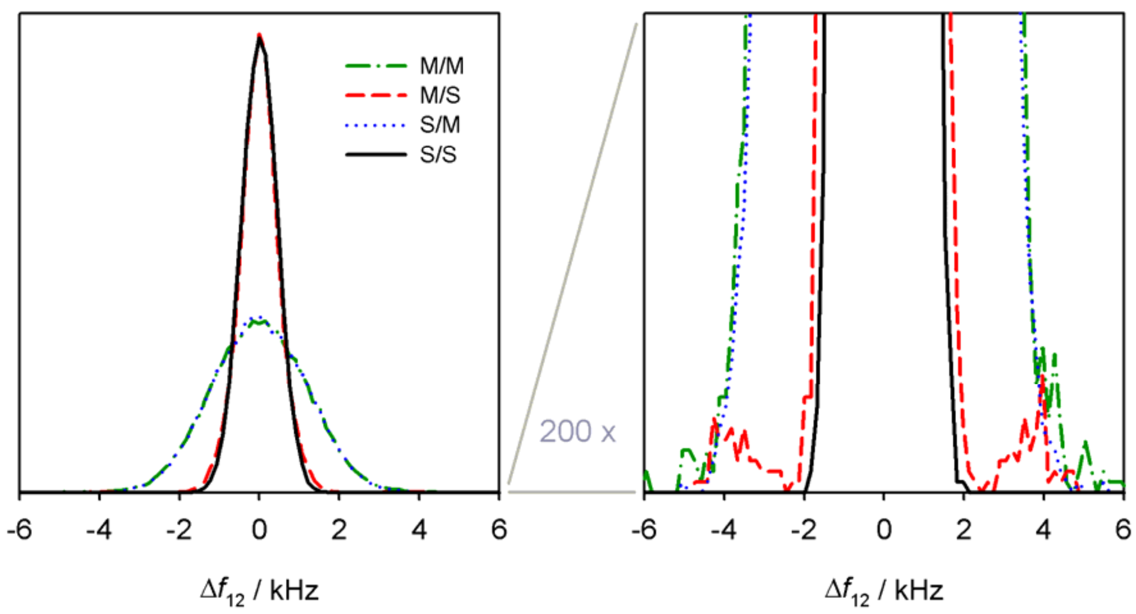

Figure 4: Histograms for 1 second samples of Δf_{12} when the repetition frequency is measured (M) or servo controlled (S). First comb is CMI Ti:Sa, second BEV fibre, respectively. In the right hand side of the picture the ordinate is enlarged by 200 times to demonstrate small differences in the side wings.

Table 4: Numerical data for the plots of Fig 4. For symbols see text.

CMI Ti:Sa	BEV fibre	σ_{1s} / Hz	σ_{1s}^{A} / Hz	$\overline{\Delta f_{12}}$ / Hz
M	M	1189	1458	+0.6
M	S	512	623	+0.8
S	M	1166	1405	+0.5
S	S	450	485	+0.7

It should be emphasized that a deconvolution of the per comb contributions is not possible within our scheme. This is a consequence of the extensive use of correlation which on the other hand enables the method described here. Nevertheless the worse performance of comb 2 (BEV Ti:Sa comb) as compared to the CMI Ti:Sa in counting mode (M) is obvious. An explanation is difficult without specialized investigations (only part of the noise can be explained from the 1 mHz resolution of the counters). On the other hand knowledge of the cause for this behaviour is not necessary for the application. One has simply to use the obtained uncertainty when the comb is operated in this mode. We want to stress out that this behaviour would be difficult to detect with other means.

4 Full comb measurement uncertainty budget

With the findings of the preceding sections the creation of an uncertainty budged (without the laser to be calibrated) is now quite straightforward. We demonstrate this using the comb BEV fibre referenced to the H-maser (BEV 40 3452).

According to (5) ρ gives the deviation of the input frequency in Hz. It has been evaluated by the time and frequency section of BEV. There are three main contributions: the uncertainty of the Cs master clock relative to TAI as obtained from circular T, the maximum deviation of H-maser relative to master clock before steering takes place (dominating influence), and the short time frequency stability of the H-maser.

Table 5: Uncertainty budged for comb based laser calibration for different integration times. Relative values are for 633 nm laser radiation.

τ	10 s	100 s	1 000 s	10 000 s
$\sigma_\tau = u(\gamma)$	75.7 Hz	8.47 Hz	1.61 Hz	0.364 Hz
$\left(f_{\text{Laser}}^{\text{N}}/f_0\right)u(\rho)$	38.3 Hz	33.9 Hz	33.8 Hz	33.8 Hz
U_{comb}	169.7 Hz	69.9 Hz	67.7 Hz	67.6 Hz
$U_{\text{comb}}/f_{\text{Laser}}^{\text{N}}$	$3.6\cdot10^{-13}$	$1.5\cdot10^{-13}$	$1.4\cdot10^{-13}$	$1.4\cdot10^{-13}$

For integration times longer than about 100 s the comb contribution $u(\gamma)$ becomes insignificant compared to the uncertainty of the used frequency reference.

From (9) it is clear that the values for σ_τ are upper bounds for $u(\gamma)$ since it includes the contribution from the second comb and $u(\delta)$, respectively. In the absent of other information one must however take this value for the uncertainty budged. ([3] clause 4.1.6).

5 Conclusion

We presented a simple experimental technique for the evaluation of the measurement uncertainty for comb based laser calibrations. The main idea in the proposed scheme is the comparison of two combs while they calibrate a laser frequency.

We found an upper bound of the uncertainty contribution originating from the comb system (including evaluation procedure, excluding frequency reference) which was in the order of a few parts in 10^{-15} for integration times 2000 s or longer. This seems to be surprisingly high as compared to results of other groups e.g. [2]. But these uncertainties refer to different measurands: multiplication of reference signal to the optical domain versus the difference of two optical clockworks, respectively.

The experiments presented here led to significant improvement of CMCs for both NMIs involved. Moreover contributions regarding tracking capability of repetition rate servos and counting capability were evaluated which would be very difficult to detect without the use of the proposed scheme.

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References

[1] http://www.bipm.org/en/cipm-mra (accessed 01.09.2011)

[2] Ma L-Sh, Bi Z, Bartels A, Robertsson L, Zucco M, Windeler R, Wilpers G, Oates Ch, Hollberg L and Diddams S A 2004 Optical Frequency Synthesis and Comparison with Uncertainty at the 10^{-19} Level, *Science* **303**: 1843-1845

[3] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML 2008 Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement *JCGM* **100**:2008 (GUM 1995 with Minor Corrections)

[4] Jones D J, Diddams S A, Ranka J K, Stenz A, Windeler R S, Hall J L and Cundiff S T 2000 Carrier-Envelope Phase Control of Femtosecond Mode-Locked Lasers and Direct Optical Frequency Synthesis, *Science* **288** 635-639

[5] Holzwarth R, Udem Th, Hänsch T W, Knight J C, Wadsworth W J and Russell P St J 2000 Optical Frequency Synthesizer for Precision Spectroscopy *Phys. Rev. Lett.* **85** 2264-2267

[6] http://kcdb.bipm.org (accessed 01.09.2011)

[7] Quinn T J 2003 Practical realization of the definition of the metre, including recommended radiations of optical frequency standards (2001) *Metrologia* **40** 103-133

[8] Stone J and Egan P 2010 An Optical Frequency Comb Tied to GPS for Laser Frequency/Wavelength Calibration *J. Res. Natl. Inst. Stand. Technol.* **115** 413-431

[9] Stenger J, Schnatz H, Tamm C and Telle H R 2002 Ultraprecise Measurement of Optical Frequency Ratios *Phys. Rev. Lett.* **88** 73601

[10] Kubina P et al. 2005 Long term comparison of two fibre based frequency comb systems *Optics Express* **13** 904-909

[11] Gray J E and Allan D W 1974 A method for estimating the frequency stability of an individual oscillator *Proceedings of the 28th Annual Symposium on Frequency Control* 243-246

[12] Won-Kyu Lee et al. 2010 The uncertainty associated with the weighted mean frequency of a phase-stabilized signal with white phase noise *Metrologia* **47** 24-32

[13] Menlo Systems GmbH

[14] Kramer G and Klische W 2001 Multi-Channel Synchronous Digital Phase Recorder *Proc. 2001 IEEE IFCS* 144-151

[15] Kramer G and Klische W 2004 Extra High Precision Digital Phase Recorder *Proc. 2004 ETFT* 144-151

[16] Matus M 2003 Femtosecond comb generator as the primary length standard in Austria *Proceedings of the 11th International Metrology Congress*

[17] Balling P, Křen P 2008 Absolute frequency measurements of wavelength standards 532 nm, 543 nm, 633 nm and 1540 nm *EUROPEAN PHYSICAL JOURNAL D* **48** 3-10

[18] Matus M et al. 2011 The CCL-K11 ongoing key comparison: final report for the year 2010 *Metrologia* **48** 04001

[19] Lee Won-Kyu et al. 2007 Comparison of Fibre-Based Frequency Comb and Ti:Sapphire-Based Frequency Comb *Journal of the Optical Society of Korea* **11** 124-129